years. He thus endeavoured to elevate the masses and foster an interest in things superior to their everyday surroundings. He died at Norton, Market Drayton, on August 28, 1884.

His only child, a son, graduated at Christ Church, Oxford, but died shortly after taking holy orders.

Mr. Silver was for many years a Fellow of the Linnæan, Royal Geographical, Geological, and Meteorological Societies.

He was elected a Fellow of this Society on May 11, 1855.

ISAAC TODHUNTER was born in 1820, and was the second son of a Congregationalist minister at Rye. Passing over his boyhood we find him an assistant-master in a school at Peckham, and at the same time attending the evening classes at University College, and among others the lectures of De Morgan. Here he seems to have come under the fascination which so many of the pupils of that great teacher experienced. His admiration for that mathematician was unbounded. He obtained great distinction in the University of London, carrying off the honours at the degrees of B.A. and M.A. He afterwards entered the University of Cambridge, and became Senior Wrangler and first Smith's Prizeman in 1848.

In the same year in which he took his degree he gained the Burney Prize. According to the regulations this prize is to be awarded for an English essay, to a graduate of the University who is not of more than three years' standing from admission to his first degree. His essay was printed in 1849 under the title "The doctrine of a Divine Providence is inseparable from the belief in the existence of an absolutely perfect Creator."

Soon after taking his degree he established himself at St John's College as a mathematical tutor and lecturer, but afterwards he gave up all share in the tuition of his college and devoted himself more and more to the work of writing books.

The great work of Dr. Todhunter's life lies in the part he has taken in the education of this generation. A detailed account of the numerous educational books he has written would be too long for so slight a sketch of his life as the present. His books conduct the student from the beginning through a long course of mathematical learning. A simple list of these is a history of the labours of his life; as the dates run on we see his time filled up with correcting one edition after another.

In constructing his books, he seems to have discovered that, for the teaching of boys, novelties would be out of place. What was wanted in any subject was a short and accurate account of the things then known. The object was to put the reader as quickly as possible in possession of all the knowledge which was most likely to be useful to him afterwards. Accordingly he gives in his books a clear statement of the well-known principles of each subject, arranged in a logical order. Each step in the argument is explained at length in clear English. Nothing is assumed but what a reader should know. Every page makes it

evident how thoroughly he was keeping in mind that he was writing for beginners.

Whatever his own ideas were, his books were certainly a great success. His "Euclid" and his "Elementary Algebra" have in twenty years run through fifteen or sixteen editions. They were appreciated by the schoolmasters, and by those who had to teach these subjects. With their recommendations, the sale has grown into something enormous. His more advanced text-books, being addressed to a more limited circle of readers, could not be expected to run through so many editions; yet we find his "Differential Calculus" reaching its ninth and his "Integral Calculus" a seventh edition.

His reputation in future time will undoubtedly rest on his histories, for the fashion of elementary books will pass away, and a new generation will like a new arrangement of old things. The most important of these are (1) "A history of the progress of the Calculus of Variations during the nineteenth century:" 1861; (2) "A history of the Mathematical Theory of Probability from the time of Pascal to that of Laplace:" 1865; (3) "A history of the Mathematical Theories of Attraction and the Figure of the Earth from the time of Newton to that of Laplace:" 1873. The first of these is a continuation of Woodhouse's history of the Calculus of Variations from its origin until the close of the eighteenth century; and it has been stated that it was his admiration for this work that led him to write this history.

These books appear to the writer of this notice to be of great importance. It is a great boon to the student to have a short and clear account of what has been already done, and what remains to be accomplished in any subject. Though the third of these histories extends over two volumes of nearly five hundred pages each, yet these are not too much for so great a subject.

It is unnecessary to give a particular account of these histories, as they have now been some time before the public. But we would call the attention of those who have not yet read them to their extreme interest. As we read one of them it seems as if a new light were thrown on the subject. The difficulties of each investigator are put before us; we see how the subject advances, each discoverer adding a little, until step by step we arrive at our present state of knowledge. We see here sketched out before us the gradual growth of those modern methods which we now find so ready to our hands. Thus, in one place, Dr. Todhunter points out the first appearance of those confocal shells which play so important a part in modern works of attraction. These appear in a memoir of Maclaurin's, who introduces them in a remarkable manner without appearing to realise their importance. In another place, we find a sketch in eight pages of a memoir of Legendre's which Dr. Todhunter considers to be the foundation of all that Laplace added in the theories of attraction and the figure of the Earth to the works

of Maclaurin and Clairaut. As we read the sketch, we see the first beginning of Laplace's coefficients and a recognition of the importance of the potential. This was the commencement of a new era in mathematical physics. In a third place, the history shows us how D'Alembert, trying to find the attraction of an ellipsoid, makes it depend on a single definite integral. This result, Dr. Todhunter reminds us, is the point at which modern investigations have finally arrived. But D'Alembert, after effecting this, strangely rejects his result as inadmissible. "In his process," says Dr. Todhunter, "there is nothing wrong in principle, but he has omitted a bracket which renders his result slightly inaccurate. He gives some invalid argument against his method. Thus D'Alembert deliberately rejects one of the most important formulæ of the subject, which in fact quite supersedes a large part of his memoir. This is perhaps the strangest of all his strange mistakes." A little further on in the history, we read how Laplace values and appropriates the treasure which D'Alembert deliberately threw away.

Dr. Todhunter has contributed a few papers to the societies to which he belonged. These are chiefly historical, and seem to have risen out of the histories on which he spent so much of his time. Some are occupied with discussions on the errors into which former writers have fallen, while others contain extensions of theorems already established. To the Royal Astronomical Society he contributed two short papers, which have both appeared in the *Monthly Notices*. In the first of these he mentions what seems to be a curious contradiction between the reasoning given in props. 36 and 37 of the third book of Newton's "Principia" and the statement made in prop. 38 of the same book. In the first two Newton discusses the forces of the Sun and Moon to move the sea, and in the last he applies the same general reasoning to find the figure of the Moon as disturbed by the attraction of the Earth. Yet while the first are quite correct, the last is wrong. The interest of the paper lies in the fact that, though this mistake was pointed out, with some emphasis, as long ago as a little past the middle of the last century, yet it does not seem to have been generally noticed by commentators on the "Principia." The second paper contributed to the Royal Astronomical Society is devoted to geodesy. An arc of the meridian measured in 1752 by La Caille, an astronomer whose accuracy has a great reputation, was found to be decidedly longer than might have been expected from measurements made in the northern hemisphere. The arc has in recent times been again measured by Sir T. Maclear, whose result is more in harmony with the northern arcs. But though the excellence of Maclear's work is beyond question, yet various difficulties and contradictions remain. These are shortly summed up by Dr. Todhunter with the hope of obtaining some explanations from those engaged in the measurements. In connection with this subject we may here allude to a longer paper contributed by Dr. Todhunter to the

twelfth volume of the *Cambridge Transactions* (1871). This is practically a history of the measurement of the famous Lapland arc first by the French and afterwards by the Swedish astronomers. He says that though the accounts of these measurements are easily accessible and written by distinguished astronomers, they yet contain numerous and serious errors. The arc then measured is a small one (being less than one degree), and therefore now unimportant when compared with the large arcs which have been since measured. The interest is, therefore, chiefly historical. It was maintained by Cassini in opposition to Newton, that the polar diameter of the earth was longer than the equatorial diameter. The operations carried on in Lapland by Maupertuis showed that the length of a degree of the meridian at the arctic circle was 57,437 toises. This was about 1,000 toises larger than it should have been according to the Cassinian theory. The result astonished the French observers. It is said that they kept this very secret, in order to reflect at leisure on what had been so little expected, and to have the pleasure of bringing the first intelligence of it to Paris.

Dr. Todhunter has also written four papers connected with the theory of the Figure of the Earth. Three of these are published in the *Proceedings* of the Royal Society. The first of these (vol. xix.) is on Jacobi's theorem on the ellipsoid of relative equilibrium and on Ivory's discussion of the theorem. In the second (vol. xx.) he gives an extension of the theorem of Poisson on the attraction of spheroids. In the third (vol. xxi.) he corrects a mistake of Dahlander. In a fourth paper, in the twelfth volume of the *Cambridge Transactions* he begins with the fundamental equation to find Yn in the Figure of the Earth; he then examines the various proofs that the only admissible value of Yn is zero given by Legendre, Laplace, O'Brien, &c., and points out the objections to each. He then concludes with a proof of his own.

We have occupied so much space with remarks on those papers of Dr. Todhunter which are more nearly connected with astronomy, that we have no room for more than an allusion to his other writings. There is a paper in the eleventh volume of the *Cambridge Transactions* on the method of Least Squares, which was evidently suggested by his history of the Mathematical Theory of Probability. In two papers in the *Proceedings* of the Royal Society for 1875 and 1878 he discusses the value of the integral of a product, first, of two, and next, of three of Legendre's functions.

In 1869, the subject prescribed for the Adams' Prize was "A determination of the circumstances under which Discontinuity of any kind presents itself in the solution of a problem of Maximum or Minimum in the Calculus of Variations." The proposal of this subject seems to have arisen from a controversy which had been carried on in the *Philosophical Magazine* a few years previously. In this controversy Dr. Todhunter had taken part, and

when the subject was proposed for the essay he was anxious that his own view should prevail. This view is given in the opening sentences of his essay:—"We shall find that, speaking generally, discontinuity is introduced by virtue of some restriction which we impose, either explicitly or implicitly, in the statement of the problems which we propose to solve." This thesis he supports by considering in turn the usual applications of the calculus, and pointing out where he considers the discontinuities which occur to have been introduced into the conditions of the problem. This he successfully proves in many instances. In some cases, the want of a distinct test of what discontinuity is, somewhat obscures the argument. His essay was rewarded with the prize. It is published under the title "Researches in the Calculus of Variations."

In the midst of so busy a life, Dr. Todhunter could yet find time to write for others. The second edition of Boole's "Differential Equations" was published under his care; and, what is more, he undertook the labour of arranging and editing the supplementary volume. This task was undertaken from friendship to the late Professor Boole. The difficulty of preparing unfinished papers for the press is obvious; and it is not surprising that, as he once mentioned to the writer, it should have cost him some months of hard work.

Dr. Todhunter has left a treatise on Elasticity, which was very nearly finished. The time and labour he spent over this work injured his eyesight, and probably led to his final illness. These MSS. had been sent to Professor Cayley to report on; and we learn from Professor Mayor's pamphlet that the investigation shows that they are of the same class as the history of the Theory of Attraction, and seem fairly complete.

Another result of the labours of his latter years is a treatise on Arithmetic. Such a work when perfected would have smoothed the way for the young beginner over many difficulties. It is greatly to be regretted that he did not undertake it sooner.

In the summer of 1880 Dr. Todhunter first began to suffer from his eyesight, and from that date he gradually and slowly became weaker. But it was not until September 1883, when he was at Hunstanton, that the worst symptoms came on. He then partially lost by paralysis the use of the right arm, and, though he afterwards recovered from this, he was left much weaker. In January of the next year he had another attack, and on March 1, 1884, he died at his own residence in Cambridge, surrounded by his dearest relatives. It was a fit ending to an honourable and respected life spent in the advancement of that science which he loved so well.

He was a member of many learned societies. He became a Fellow of the Royal Society in 1862, and served on the Council during the years 1871–1873. He was elected a member of the Mathematical Society of London in 1865, the first year of its

existence. He was elected a Fellow of this Society on January 11, 1850, and was a member of the Council during the two years from February 1868 to February 1870. Lastly, when the University of Cambridge established its new degree of Doctor of Science, restricted to those who have made original contributions to the advancement of science or learning, he was one of those whose application was granted within the first few months.

E. J. R.

THOMAS TURNER was born at Hull, December 13, 1804, and was the third son of Ralph Turner, a merchant of that place. He was educated at Bingley School, in Yorkshire, under the Rev. Dr. Hartley, and subsequently under the tutelage of the Rev. C. Bird, late Canon of Lincoln.

In 1823, he entered at Trinity College, Cambridge, of which he became a scholar and afterwards Fellow, having been second Wrangler and first Smith's Prizeman in 1827.

After taking his degree, he for a time took pupils, amongst whom was the late Mr. Milner Gibson. He then took up the study of law, and was called to the bar as a member of the Society of Lincoln's Inn. He practised as a barrister in London till 1846, when he retired from practice and devoted himself to philanthropic works. Upon the consecration of his valued friend, the Rev. Charles Perry, to the bishopric of Melbourne, he took much interest in the affairs of that diocese, and by his legal knowledge was able to give valuable assistance in obtaining the assent of the Queen to the act of the Victorian Parliament for the constitution of a Church of England Legislature in the colony.

About the same time also he was able to give much help to the people of Manchester, through his connection with Mr. Henry Charlewood, in obtaining the "Parish of Manchester Division Act," 1850, whereby the parochial duties of the dean and canons, and the disposal of the revenues were regulated, and provision was made for the division of the ancient parish of Manchester into separate rectories. Great research was necessary, and the effect of various charters had to be considered. To this difficult subject Mr. Turner devoted his time and talents, and to the assistance he gave the comprehensiveness and value of that Act was largely due.

About 1848 he was made a magistrate for the county of Middlesex, and was for several years chairman of the Parliamentary Committee, besides being an active member of several other committees. He was subsequently made a deputy-lieutenant of the same county.

In 1855, on the passing of Sir B. Hall's Local Government Act, he was elected as a member of the Metropolitan Board of Works, for Hampstead, and later on, after his removal to Southwark, he became a member of the Board, for St. Olave's.

In 1856, Mr. Turner undertook the office of treasurer to